

## MTH/MATH 253Z Calculus: Sequences and Series

For more detailed information, see CCN Reports & Memos on the [Resources for CCN](#) webpage.

### CCN Course/Course Information

#### Math

**Course Number and Prefix:** MTH or MATH 253Z

**Course Title:** Calculus: Sequences and Series

**Course Credits:** 4

**Course Description:** This course explores real-valued sequences and series, including power and Taylor series. Topics include convergence and divergence tests and applications. These topics will be explored graphically, numerically, and symbolically. This course emphasizes abstraction, problem-solving, reasoning, communication, connections with other disciplines, and the appropriate use of technology.

#### Course Learning Outcomes:

At the end of the course, students will be able to...

1. Recognize and define sequences in a variety of forms and describe their properties, including the concepts of convergence and divergence, boundedness, and monotonicity.
2. Recognize and define series in terms of a sequence of partial sums and describe their properties, including convergence and divergence.
3. Recognize series as harmonic, geometric, telescoping, alternating, or  $p$ -series, and demonstrate whether they are absolutely convergent, conditionally convergent, or divergent, and find their sum if applicable.
4. Choose and apply the divergence, integral, comparison, limit comparison, alternating series, and ratio tests to determine the convergence or divergence of a series.
5. Determine the radius and interval of convergence of power series, and use Taylor series to represent, differentiate, and integrate functions.
6. Use techniques and properties of Taylor polynomials to approximate functions and analyze error.

#### Required Course Content:

At the end of the course, students will be able to...

1. Recognize and define sequences in a variety of forms and describe their properties, including the concepts of convergence and divergence, boundedness, and monotonicity.
  - a. Students will be able to define and recognize sequences given explicitly or recursively.
  - b. Students will be able to determine whether a given sequence is convergent or divergent by appropriate use of the limit laws for sequences, the Squeeze Theorem, or L'Hôpital's rule.
  - c. Students will be able to determine the monotonicity and boundedness properties of a sequence and use them to draw conclusions about convergence or divergence.
2. Recognize and define series in terms of a sequence of partial sums and describe their properties, including convergence and divergence.
  - a. Students will be able to represent a series as a limit of a sequence of partial sums and describe the notions of convergence or divergence of the series.
  - b. Students will be able to algebraically manipulate series, and apply series laws to draw conclusions about divergence, convergence, and the value of the limit.
3. Recognize series as harmonic, geometric, telescoping, alternating, or  $p$ -series, and demonstrate whether they are absolutely convergent, conditionally convergent, or divergent, and find their sum if applicable.

4. Choose and apply the divergence, integral, comparison, limit comparison, alternating series, and ratio tests to determine the convergence or divergence of a series.
  - a. Students will be able to recognize when the divergence, integral, comparison, and limit comparison tests apply to a particular series, and draw conclusions about the convergence or divergence of the series.
  - b. Students will be able to recognize when the ratio and alternating series tests apply to a particular series, and draw conclusions about the absolute convergence, conditional convergence, or divergence of a series.
5. Determine the radius and interval of convergence of power series, and use Taylor series to represent, differentiate, and integrate functions.
  - a. Students will be able to find the radius and interval of convergence of a given power series.
  - b. Students will be able to use power series to represent functions and determine the radius of convergence of the series.
  - c. Students will be able to differentiate and integrate power series that represent functions.
  - d. Students will be able to find the Taylor series centered at a point  $x=c$  of a given function and determine its radius of convergence.
6. Use techniques and properties of Taylor polynomials to approximate functions and analyze error.
  - a. Students will be able to approximate a function using a Taylor polynomial.
  - b. Students will be able to estimate the error in a Taylor polynomial approximation using either Taylor's Inequality or the Alternating Series Estimation Theorem.
  - c. Students will be able to approximate an alternating series to a desired error by a partial sum of the series.